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Aircraft Vulnerable-Area Decomposition Method in the Overlapping Region of Components

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At present, the precise calculation methods, namely, Markov chain or tree diagram, for aircraft survivability or vulnerability to multiple hits by nonexplosive penetrators cannot deal with the situation where the components overlap in an arbitrary manner. This paper, based on probability theory and the analysis of the kill states of aircraft and components, proposes a method for decomposing the vulnerable area in the components' overlapping region. After vulnerable-area decomposition, the overlapping region becomes several individual "broadly vulnerable components" with different kill properties, and then the multiple-hit vulnerability computation of aircraft with multiple redundant sets and arbitrary overlapping among components can be proceeded utilizing Markov chain or tree diagram method. The examples provide the computation and analysis of the cumulated kill probability of aircraft exposed to the threat of multiple hits and demonstrate the general applicability and feasibility of the proposed vulnerable-area decomposition method in the components' overlapping region.

I. Introduction

AIRCRAFT combat survivability (ACS)[1] is defined as the capability of an aircraft to avoid or withstand a man-made hostile environment. Survivability is composed of two focus areas, which are susceptibility and vulnerability. One of the important parts in aircraft vulnerability assessment is to calculate the cumulated kill probability of aircraft subjected to the multiple hits by nonexplosive penetrators (multiple-hit vulnerability). At present, two precise calculation methods, namely, Markov chain or tree diagram, are commonly used to analyze the aircraft multiple-hit vulnerability.[1–5] But both of them cannot deal with the computation of the cumulated probability of kill in the situation where the components overlap in an arbitrary manner. When components overlap, the overlapping region hit by one penetrator will sometimes produce different component kill states with different kill properties. For example, when two nonredundant components (component *a* and component *b*) overlap, the overlapping area *ab* hit by one penetrator will produce three component kill states: only component *a* killed, only component *b* killed, and components *a* and *b* both killed. Because the two components are both nonredundant, the three kill states will lead to an aircraft kill, which means the three component kill states have the same kill property. However, when the two components are mutually redundant, only kill of both of them can lead to an aircraft kill. In this case, the three component kill states have totally different kill properties, which implies that only kill of component *a* will make component *b* become a nonredundant component in the overlapping region, that only kill of component *b* will make component *a* become a nonredundant component in the overlapping region, and that kill of both components *a* and *b* will result in kill of the aircraft.

In the actual aircraft combat operation, the hit aspect of penetrator is arbitrary, so that the overlapping regions among components appear very widely. The preceding example shows that the overlapping region hit by one penetrator will usually produce several component kill states with entirely different kill properties. Hence the Markov chain or tree diagram in aircraft multiple-hit vulnerability cannot be proceeded because the overlapping region cannot be treated as an individual "broadly vulnerable component." This paper, based on probability theory and the analysis of the kill states of aircraft and components, proposes a method for decomposing the vulnerable area in the components' overlapping region. After vulnerable-area decomposition, the overlapping region becomes several individual broadly vulnerable components with different kill properties, and then the multiple-hit vulnerability computation of aircraft with multiple redundant sets and arbitrary overlapping among components can be proceeded utilizing Markov chain or tree diagram method.

II. Assumptions

Aircraft multiple-hit vulnerability is based on the single-hit vulnerability. The vulnerability of the aircraft for a particular threat aspect is usually expressed as the probability the aircraft is killed given a random (uniformly distributed) hit anywhere on the presented area of the aircraft_$P_{K/H}$, or as the vulnerable area A_V. A_V is related to $P_{K/H}$ by[1]

$$A_V = A_P P_{K/H} \tag{1}$$

where A_P denotes the projected area of aircraft in the plane normal to the approach direction of threat. The computation of $P_{K/H}$ and A_V is based on the component's probability of kill and vulnerable area given a hit. The vulnerable area of the ith component A_{v_i} is defined as

$$A_{v_i} = A_{p_i} P_{k/h_i} \tag{2}$$

where P_{k/h_i} is the probability the ith component is killed given a hit on the ith component and A_{p_i} denotes the presented area of the ith component. When components overlap, formula (2) should be modified, in which A_{p_i} and P_{k/h_i} are the presented area and probability of kill given a hit in the overlapping area, respectively. This paper is based on the following assumptions:

1) The A_{p_i} and P_{k/h_i} of each component and aircraft presented area A_P are known. Please refer to Refs. 1 and 6 for the calculation methods for A_{p_i} and P_{k/h_i}.

2) Component when hit has only two states, namely, kill or no kill.

3) Component kills caused by a hit in an overlap region are independent outcomes; a kill of one component has no effect on the kill of another component.

4) The notation used in this paper is described as follows. For component x, the kill event is denoted by X, whereas the no kill

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event is denoted by $\bar{X}$. P_{k/h_x} denotes the probability of kill given a hit on component x.

5) Accounting for the effects of shielding of one component by another, the values of P_{k/h_i} for overlapping components are affected by shielding.

III. Determining the Kill Properties of the Components in Overlapping Region

In the precise calculation methods for aircraft multiple-hit vulnerability, the determination of the number of tree branches in tree diagram and the dimension of state transition matrix in Markov chain are both based on the kill properties of components. As already mentioned, the overlapping region among components hit by one penetrator will usually produce several component kill states with entirely different kill properties, and so we first analyze the kill properties of the components in the overlapping region. When two components overlap, the overlapping region can be divided into three categories. Any complex overlapping region among components can be regarded as the combination of the three cases: 1) overlap between nonredundant component and nonredundant component, 2) overlap between nonredundant component and redundant component, and 3) overlap between redundant component and redundant component.

In the following, we illustrate the determination of kill properties of the components in the just-mentioned three cases by analyzing the overlapping region *ab* of components a and b in Fig. 1. Then a general determination of kill properties of the components in the overlapping region will be proposed. In Fig. 1, the overlapping region hit by a threat will produce three component kill states: 1) only component a has been killed, namely, $A\bar{B}$; 2) only component b has been killed, namely, $\bar{A}B$; and 3) both component a and component b have been killed, namely, AB.

A. Overlap Between Nonredundant Components

Suppose both component a and component b are nonredundant, which implies that kill of any of them will result in an aircraft kill. Hence, the three component kill states (1, 2, and 3) have the same kill property. When computing the multiple-hit vulnerability using Markov chain or tree diagram method, overlapping region *ab* can be treated as a broadly vulnerable component (nonredundant component).

B. Overlap Between Nonredundant Component and Redundant Component

Suppose component a is nonredundant and component b is one of the redundant components in a redundant set. In this case, states 1 and 3 have the same kill property, which will result in an aircraft kill. When computing the multiple-hit vulnerability using Markov chain or tree diagram method, in order to reduce the number of tree branches or the dimension of state transition matrix, states 1 and 3 can be merged into one kill state, namely, the kill of component a ($A\bar{B} \cup AB = A$).

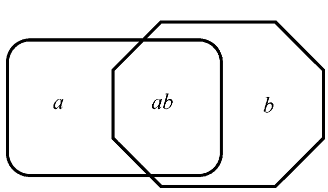

Fig. 1 Overlapping area of components *a* and *b*.

C. Overlap Between Redundant Components

Suppose components a and b are both redundant, then the properties of the three kill states are entirely different. Kill states 1 and 2 will not result in kill of the aircraft. Whether kill state 3 will lead to an aircraft kill or not, it depends on the property of the redundant components that overlap. When and only when components a and b are mutually redundant, kill state 3 will result in kill of the aircraft.

D. Overlap Among More Than Two Components

As is already mentioned, any complex overlapping region among components can be regarded as the combination of the three cases. When the number of overlapping components is more than two, the process of determination of component kill states is the same as the one when two components overlap as already mentioned. The main difference is that the number of component kill states will increase with the increase of the number of overlapping components. However, because kill of the nonredundant components will result in kill of the aircraft, the number of kill properties totally depends on the set number of redundant components and the number of components in each redundant set and is related to the number of combinations of kill states of redundant components.

IV. Vulnerable-Area Decomposition Method in the Overlapping Region

After decomposition of the kill properties in the components' overlapping region, the next step will be the determination of the vulnerable area corresponding to each kill property, which is also called vulnerable-area decomposition in the components' overlapping region. Based on probability theory, according to the preceding analysis of the kill properties, the vulnerable-area decomposition method for the overlapping region of components a and b is shown in Table 1. In Table 1, P_{k/h_a} and P_{k/h_b} refer to the probability of kill given a hit on component a and component b, respectively, and A_{po} is the presented area of the overlapping region. The values of $P_{k/h}$ for overlapping components are affected by shielding.

The preceding section has described the analysis of kill properties and the vulnerable-area decomposition method for the three overlapping cases of two components. In actual situations the three cases usually combine in an arbitrary manner. However, the vulnerable-area decomposition method in the overlapping region is the same, which includes the following steps. First, list all of the component kill states according to the combinations of the redundant components in the overlapping region. And then identify the kill property of each kill state, and merge the kill states according to their kill properties. Finally, calculate the vulnerable area corresponding to each individual kill property.

For example, assume one overlapping region of an aircraft consists of N_0 nonredundant components $(x_1, x_2, \ldots, x_{N0})$ and L redundant sets. The ith redundant set is denoted by Y_i and composed of n_i components. The overlapping region has m_i components of the ith redundant set. Let y_{ij} be the jth component in the ith redundant set, and $P_{k/h_{yi}}$ be the probability of kill of the ith redundant set given a hit in the overlapping region. $P_{k/h_{yi}}$ can be calculated according to the "kill tree"[1] for the aircraft. Kill tree is a visual illustration of the critical components and all component redundancies. The kill tree

Table 1 Vulnerable-area decomposition method in the overlapping region of two components

Overlapping case	Kill property	Corresponding vulnerable area
Overlap beween nonredundant components	Nonredundant components killed (aircraft killed): $A \cup B$	$A_{po}\left[P_{k/h_b} + P_{k/h_a}\left(1 - P_{k/h_b}\right)\right]$
Overlap between nonredundant component and redundant component	Component a killed (aircraft killed): A	$A_{po} P_{k/h_a}$
	Only component b killed (aircraft survives): $\bar{A}B$	$A_{po}\left(1 - P_{k/h_a}\right) P_{k/h_b}$
Overlap between redundant components	Only component a killed (aircraft survives): $A\bar{B}$	$A_{po} P_{k/h_a}\left(1 - P_{k/h_b}\right)$
	Only component b killed (aircraft survives): $\bar{A}B$	$A_{po}\left(1 - P_{k/h_a}\right) P_{k/h_b}$
	Both components killed (aircraft survives or killed): AB	$A_{po} P_{k/h_a} P_{k/h_b}$

Table 2 Vulnerable-area decomposition method for the overlapping region of more than two components

Kill event	Corresponding vulnerable area
Nonredundant components or redundant set(s) killed (aircraft killed) $(X_1 U X_2 U \cdots U X_{N0} U Y_1 U Y_2 U \cdots U Y_L)$	$A_{\mathrm{po}}\left\{\sum_{q=1}^{N_0}\left[P_{\mathrm{k/h}_q}\prod_{i=1}^{q-1}\left(1-P_{\mathrm{k/h}_i}\right)\right]+\prod_{w=1}^{N_0}\left(1-P_{\mathrm{k/h}_w}\right)\cdot\sum_{r=1}^{L}\left[P_{\mathrm{k/h_{yr}}}\prod_{s=1}^{r-1}\left(1-P_{\mathrm{k/h_{ys}}}\right)\right]\right\}$
Only the $i1$th component killed in Ω, and aircraft survives at event U_{i1}	$A_{\mathrm{po}}\prod_{j=1}^{N_0}\left(1-P_{\mathrm{k/h}_j}\right)\cdot\prod_{q=1}^{1}P_{\mathrm{k/h_{u(iq)}}}\cdot\prod_{r=1,\ r\neq i1}^{R}\left(1-P_{\mathrm{k/h_{ur}}}\right),\ (\forall 1\le i1\le R)$
Only the $i1$th and $i2$th components killed in Ω, and aircraft survives at event $U_{i1}U_{i2}$	$A_{\mathrm{po}}\prod_{j=1}^{N_0}\left(1-P_{\mathrm{k/h}_j}\right)\cdot\prod_{q=1}^{2}P_{\mathrm{k/h_{u(iq)}}}\cdot\prod_{r=1,\ r\neq i1,i2}^{R}\left(1-P_{\mathrm{k/h_{ur}}}\right),\ (\forall 1\le i1< i2\le R)$
Only the $i1$th, $i2$th, and $i3$th components killed in Ω, and aircraft survives at event $U_{i1}U_{i2}U_{i3}$	$A_{\mathrm{po}}\prod_{j=1}^{N_0}\left(1-P_{\mathrm{k/h}_j}\right)\cdot\prod_{q=1}^{3}P_{\mathrm{k/h_{u(iq)}}}\cdot\prod_{r=1,\ r\neq i1,i2,i3}^{R}\left(1-P_{\mathrm{k/h_{ur}}}\right),\ (\forall 1\le i1< i2< i3\le R)$
Only the $i1$th, $i2$th, ..., and $i(R-1)$th components killed in Ω, and aircraft survives at event $U_{i1}U_{i2}\cdots U_{i(R-1)}$	$A_{\mathrm{po}}\prod_{j=1}^{N_0}\left(1-P_{\mathrm{k/h}_j}\right)\cdot\prod_{q=1}^{R-1}\mathrm{P}_{\mathrm{k/h_{u(iq)}}}\cdot\prod_{r=1,\ r\neq i1,i2,\ldots,i(R-1)}^{R}\left(1-P_{\mathrm{k/h_{ur}}}\right),\ [\forall 1\le i1<\cdots< i(R-1)\le R]$
Only the $i1$th, $i2$th, ..., and iRth components killed in Ω, and aircraft survives at event $U_{i1}U_{i2}\cdots U_{i(R-1)}U_{iR}$	$A_{\mathrm{po}}\prod_{j=1}^{N_0}\left(1-P_{\mathrm{k/h}_j}\right)\cdot\prod_{q=1}^{R}P_{\mathrm{k/h_{u(iq)}}},\ (i1=1, i2=2,\ldots, iR=R)$

illustrates the number of redundant components in a redundant set that must be killed to cause an aircraft kill.

Assuming that kill of all of the components of each redundant set lead to kill of the redundant set, then $P_{\mathrm{k/h_{yi}}}$ can be expressed as

$$P_{\mathrm{k/h_{yi}}}=\begin{cases}0 & (m_i<n_i)\\ \prod_{p=1}^{n_i}\mathrm{P}_{\mathrm{k/h_{yip}}} & (m_i=n_i)\end{cases} \tag{3}$$

where $P_{\mathrm{k/h_{yip}}}$ is the probability of kill given a hit on y_{ip}, the pth component in the ith redundant set. Let Ω be the set containing all of the redundant components in the overlapping region and denoted by $\Omega=(u_1,u_2,\ldots,u_R)$, where R is the number of elements in Ω, and

$$R=\sum_{i=1}^{L}m_i \tag{4}$$

Table 2 shows the vulnerable-area decomposition method for the overlapping region of more than two components, where $P_{\mathrm{k/h_{ui}}}$ is the probability of kill of component u_i in Ω given a hit in the overlapping region. From the table, one can see that the first event in the "kill event" column can cause an aircraft kill. However, the aircraft survives at all of the remaining evens. The second event in the kill event column refers to only one redundant component killed in Ω. The third event in the kill event column refers to only two redundant components killed in Ω, and so on.

V. Multiple-Hit Vulnerability Assessment Based on the Vulnerable-Area Decomposition

The preceding analysis reveals that after vulnerable-area decomposition the overlapping region becomes several individual broadly vulnerable components with different kill properties. After the decomposition is complete, the multiple-hit vulnerability computation of aircraft with multiple redundant sets and arbitrary overlapping among components can be proceeded utilizing Markov chain or tree diagram method. When applying the vulnerable decomposition method to the multiple-hit vulnerability assessment using Markov chain method, the key lies in the consideration of the individual kill states of the overlapping area when constructing the state transition matrix. When applying the vulnerable decomposition method to the multiple-hit vulnerability assessment using tree diagram method, the key lies in the consideration of the individual tree branches of the overlapping region. Markov chain and tree diagram methods calculate the cumulated probability of kill of aircraft progressively according to the sequence of the threat hits, and more details can be found in Ref. 1.

Table 3 Assumed values for A_P and $P_{\mathrm{k/h}}$

Critical component	A_P, m^2	$P_{\mathrm{k/h}}$
Nonoverlapping		
a	20	1.0
b	40	0.4
Overlapping		
a	30	0.2 (shielding effect)
b	30	0.4

VI. Examples

This section contains two examples. In example 1, a simple problem involving the overlapping of two components is used as an illustration of the general applicability of the method. Example 2 gives the multiple-hit vulnerability assessment of an aircraft containing eight critical components at the right threat hits. Example 2 also compares the results from the Markov chain method for hits 1–10 with the results from the simplified approach.[1]

A. Example 1

As a numerical illustration, assume an aircraft contains two mutually redundant components (a and b) with component b directly in front of component a for the given threat aspect, as is shown in Fig. 1. And assume $P_{\mathrm{k/h}_a}=1.0$ and $P_{\mathrm{k/h}_b}=0.4$ for the combination of threat mass and velocity of interest. Accounting for the effects of shielding in the overlapping region, the $P_{\mathrm{k/h}}$ for component a reduces to 0.2. The assumed values for the component presented area and $P_{\mathrm{k/h}}$ are shown in Table 3. The A_P of aircraft is 100 m^2 for this example.

According to the proposed method, the process of multiple-hit vulnerability assessment using Markov chain method can be divided into the following four steps.

1. List all of the Component Kill States and the Corresponding Vulnerable Areas

Table 4 shows all the component kill states and their corresponding vulnerable areas in overlapping and nonoverlapping regions.

2. Merge the Component Kill States According to Kill Properties

Table 5 shows the individual component kill states and their corresponding vulnerable areas.

3. Construct the State Transition Matrix of Markov Chain

According to Table 5, the aircraft can exist in four distinct states:

1) Both component a and component b have been killed, resulting in an aircraft kill, denoted by $Knrc$.

2) Only component a has been killed, denoted by a.

3) Only component b has been killed, denoted by b.

4) None of the critical components are killed, denoted by nk. The corresponding area of this state is 100.0-23.6-25.6-2.4 = 48.4 m^2.

The state transition matrix of probability [$\boldsymbol{T}$] that specifies how the aircraft will transition from one state to another as a result of a hit on the aircraft is constructed as shown in Table 6, in which the element $\boldsymbol{T}_{ij}$ represents the probability of transitioning from the state i by the column location to the new state j by the row location.[1]

4. Computing the Multiple-Hit Vulnerability

According to state transition matrix of probability [$\boldsymbol{T}$], the state vector $\{\boldsymbol{S}_i\}$ consisting of the probabilities that the aircraft is in each of the four states after the ith hit is given by[1]

$$\{\boldsymbol{S}_i\} = [\boldsymbol{T}]\{S_{i-1}\} \tag{5}$$

where $\{\boldsymbol{S}_0\}$ denotes the initial state vector prior to the first hit

$$\{\boldsymbol{S}_0\} = \{0.0, 0.0, 0.0, 1.0\}$$

After matrix multiplications given by Eq. (5), the values of cumulated kill probability $P_{\mathrm{K/HN}}$ for N hits ($N = 1, 2, \ldots, 10$) using the Markov chain method are shown in Table 7.

B. Example 2

Using the multiple-hit vulnerability assessment of an aircraft pictured at the left of Fig. 2 as an example, we first decompose the vulnerable area in the overlapping region using the proposed

Table 5 Individual component kill states and the corresponding vulnerable areas

Individual kill state	Corresponding vulnerable area, m^2
Only component a has been killed	$20.0 + 3.6 = 23.6$
Only component b has been killed	$16.0 + 9.6 = 25.6$
Both component a and component b have been killed (aircraft killed)	2.4

Table 6 State transition matrix of probability [T] of example 1

	Probability of transitioning from this state				
$1/A_P$	*Knrc*	a	b	nk	To this state
1/100.0	100.0	(25.6 + 2.4)	(23.6 + 2.4)	2.4	*Knrc*
	0.0	(23.6 + 48.4)	0.0	23.6	a
	0.0	0.0	(25.6 + 48.4)	25.6	b
	0.0	0.0	0.0	48.4	nk

Table 7 Values for $P_{\mathrm{K/HN}}$

N	$P_{\mathrm{K/HN}}$
1	0.024000
2	0.168256
3	0.334908
4	0.486272
5	0.611168
6	0.709334
7	0.784403
8	0.840872
9	0.882918
10	0.914027

Table 4 Component kill states and the corresponding vulnerable areas

Region	Kill state	Corresponding vulnerable area, m^2
Nonoverlapping	(Only) component a has been killed	$20 \times 1.0 = 20.0$
	(Only) component b has been killed	$40 \times 0.40 = 16.0$
Overlapping	Only component a has been killed	$30 \times 0.20 \times (1 - 0.4) = 3.6$
	Only component b has been killed	$30 \times 0.40 \times (1 - 0.2) = 9.6$
	Both component a and component b have been killed (aircraft killed)	$30 \times 0.20 \times 0.40 = 2.4$

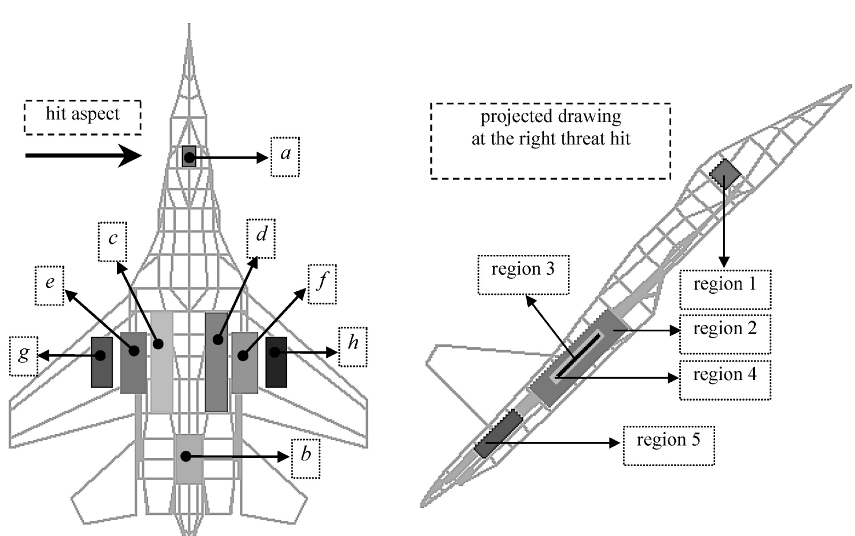

Fig. 2 Aircraft vulnerable decomposition example.

method and then apply the vulnerable-area decomposition method to the multiple-hit vulnerability assessment utilizing Markov chain method. To compare the results from Markov chain method with the results from simplified approach,[1] the effects of shielding of one component by another are not considered, and the values of $P_{\mathrm{k/h}}$ for overlapping components are not affected by shielding.

1. *Vulnerable-Area Decomposition*

The left portion of Fig. 2 shows the planform of the aircraft containing eight critical components. Components a, b, c, and d are nonredundant, components e and f are mutually redundant in the first redundant set, and components g and h are mutually redundant in the second redundant set. The probability of kill given a hit on the component $P_{\mathrm{k/h}}$ for every component is $P_{\mathrm{k/h}_a} = 1.00$, $P_{\mathrm{k/h}_b} = 0.40$, $P_{\mathrm{k/h}_c} = 0.25$, $P_{\mathrm{k/h}_d} = 0.20$, $P_{\mathrm{k/h}_e} = 0.50$, $P_{\mathrm{k/h}_f} = 0.45$, $P_{\mathrm{k/h}_g} = 0.35$, and $P_{\mathrm{k/h}_h} = 0.3$. The right portion of Fig. 2 shows the projected drawing at the right threat hit. At this threat aspect, the presented area of the aircraft Ap is 55.57 m^2 and there form five critical regions among components. The main characteristics about the five regions are shown in Table 8.

Table 8 Aircraft projected region description at the given threat aspect

Region number	Characteristic description	Corresponding A_{po}, m^2
1	Only the projected region of component a and no overlapping	0.64
2	Overlapping between components c and d	3.00
3	Overlapping among components g, e, c, d, f, and h	0.20
4	Overlapping among components e, c, d, and f	0.80
5	Only the projected region of component b and no overlapping	1.00

Table 9 shows the vulnerable decomposition results in the critial regions (including overlapping and nonoverlapping regions) where

$$P_{\mathrm{k/h}_{y1}} = P_{\mathrm{k/h}_e} \qquad P_{\mathrm{k/h}_f} = 0.50 * 0.45 = 0.225$$

and

$$P_{\mathrm{k/h}_{y2}} = P_{\mathrm{k/h}_g} \qquad P_{\mathrm{k/h}_h} = 0.30 * 0.35 = 0.105$$

According to Table 9, we can see that the aircraft can exist in 10 distinct states (see Table 10). Through merging the corresponding vulnerable areas of each kill event in each region, we can get the total vulnerable area of each state shown in Table 10.

2. *Computing the Multiple-Hit Vulnerability Using Markov Chain Method*

After the vulnerable-area decomposition is complete, the state transition matrix of probability $[\boldsymbol{T}]$ could be constructed, as is shown in Table 11.

The main character of the transition matrix is the sum of the elements of each column equal to unity. From the preceding matrix, we can see the sum of the elements of each column is equal to $(\mathrm{i} + \mathrm{ii} + \mathrm{iii} + \mathrm{iv} + \mathrm{v} + \mathrm{vi} + \mathrm{vii} + \mathrm{viii} + \mathrm{ix} + \mathrm{x})/A_P = 1$, as it should be. Substituting the numerical values in Table 10 into $[\boldsymbol{T}]$, the elements of $[\boldsymbol{T}]$ can be obtained:

Table 9 Vulnerable-area decomposition results in the critical regions

Region number	Kill event	Corresponding vulnerable area, m^2
1	Nonredundant component killed (aircraft killed)	$A_{\mathrm{po}1} P_{\mathrm{k/h}_a} = 0.64$
2	Nonredundant components killed (aircraft killed)	$A_{\mathrm{po}2}[P_{\mathrm{k/h}_c} + P_{\mathrm{k/h}_d}(1 - P_{\mathrm{k/h}_c})] = 1.2$
3	Nonredundant components or redundant sets killed (aircraft killed)	$A_{\mathrm{po}3}[P_{\mathrm{k/h}_c} + P_{\mathrm{k/h}_d}(1 - P_{\mathrm{k/h}_c}) + P_{\mathrm{k/h}_{y1}}(1 - P_{\mathrm{k/h}_c})(1 - P_{\mathrm{k/h}_d}) + P_{\mathrm{k/h}_{y2}}(1 - P_{\mathrm{k/h}_c})(1 - P_{\mathrm{k/h}_d})(1 - P_{\mathrm{k/h}_{y1}})] = 0.116765$
	Only component e killed	$A_{\mathrm{po}3} P_{\mathrm{k/h}_e}(1 - P_{\mathrm{k/h}_f})(1 - P_{\mathrm{k/h}_g})(1 - P_{\mathrm{k/h}_h})(1 - P_{\mathrm{k/h}_c})(1 - P_{\mathrm{k/h}_d}) = 0.015015$
	Only component f killed	$A_{\mathrm{po}3} P_{\mathrm{k/h}_f}(1 - P_{\mathrm{k/h}_e})(1 - P_{\mathrm{k/h}_g})(1 - P_{\mathrm{k/h}_h})(1 - P_{\mathrm{k/h}_c})(1 - P_{\mathrm{k/h}_d}) = 0.012285$
	Only component g killed	$A_{\mathrm{po}3} P_{\mathrm{k/h}_g}(1 - P_{\mathrm{k/h}_e})(1 - P_{\mathrm{k/h}_f})(1 - P_{\mathrm{k/h}_h})(1 - P_{\mathrm{k/h}_c})(1 - P_{\mathrm{k/h}_d}) = 0.008085$
	Only component h killed	$A_{\mathrm{po}3} P_{\mathrm{k/h}_h}(1 - P_{\mathrm{k/h}_e})(1 - P_{\mathrm{k/h}_g})(1 - P_{\mathrm{k/h}_f})(1 - P_{\mathrm{k/h}_c})(1 - P_{\mathrm{k/h}_d}) = 0.006435$
	Only components e and g killed	$A_{\mathrm{po}3} P_{\mathrm{k/h}_e} P_{\mathrm{k/h}_g}(1 - P_{\mathrm{k/h}_f})(1 - P_{\mathrm{k/h}_h})(1 - P_{\mathrm{k/h}_c})(1 - P_{\mathrm{k/h}_d}) = 0.008085$
	Only components e and h killed	$A_{\mathrm{po}3} P_{\mathrm{k/h}_e} P_{\mathrm{k/h}_h}(1 - P_{\mathrm{k/h}_f})(1 - P_{\mathrm{k/h}_g})(1 - P_{\mathrm{k/h}_c})(1 - P_{\mathrm{k/h}_d}) = 0.006435$
	Only components f and g killed	$A_{\mathrm{po}3} P_{\mathrm{k/h}_f} P_{\mathrm{k/h}_g}(1 - P_{\mathrm{k/h}_e})(1 - P_{\mathrm{k/h}_h})(1 - P_{\mathrm{k/h}_c})(1 - P_{\mathrm{k/h}_d}) = 0.006615$
	Only components f and h killed	$A_{\mathrm{po}3} P_{\mathrm{k/h}_f} P_{\mathrm{k/h}_h}(1 - P_{\mathrm{k/h}_e})(1 - P_{\mathrm{k/h}_g})(1 - P_{\mathrm{k/h}_c})(1 - P_{\mathrm{k/h}_d}) = 0.005265$
4	Nonredundant components or redundant sets killed (aircraft killed)	$A_{\mathrm{po}4}[P_{\mathrm{k/h}_c} + P_{\mathrm{k/h}_d}(1 - P_{\mathrm{k/h}_c}) + P_{\mathrm{k/h}_{y1}}(1 - P_{\mathrm{k/h}_c})(1 - P_{\mathrm{k/h}_d})] = 0.428000$
	Only component e killed	$A_{\mathrm{po}4} P_{\mathrm{k/h}_e}(1 - P_{\mathrm{k/h}_f})(1 - P_{\mathrm{k/h}_c})(1 - P_{\mathrm{k/h}_d}) = 0.132000$
	Only component f killed	$A_{\mathrm{po}4} P_{\mathrm{k/h}_f}(1 - P_{\mathrm{k/h}_e})(1 - P_{\mathrm{k/h}_c})(1 - P_{\mathrm{k/h}_d}) = 0.108000$
5	Nonredundant component killed (aircraft killed)	$A_{\mathrm{po}5} P_{\mathrm{k/h}_b} = 0.40$

Table 10 Aircraft existent states and corresponding vulnerable areas

State number	Kill event	Corresponding vulnerable area, m^2
I	Aircraft killed: $(A \cup B \cup C \cup D \cup EF \cup GH)$	$\mathrm{i} = 2.784765$
II	Only component e killed: $E \cap \bar{F} \cap \bar{G} \cap \bar{H} \cap \bar{A} \cap \bar{B} \cap \bar{C} \cap \bar{D}$	$\mathrm{ii} = 0.147015$
III	Only component f killed: $\bar{E} \cap F \cap \bar{G} \cap \bar{H} \cap \bar{A} \cap \bar{B} \cap \bar{C} \cap \bar{D}$	$\mathrm{iii} = 0.120285$
IV	Only component g killed: $\bar{E} \cap \bar{F} \cap G \cap \bar{H} \cap \bar{A} \cap \bar{B} \cap \bar{C} \cap \bar{D}$	$\mathrm{iv} = 0.008085$
V	Only component h killed: $\bar{E} \cap \bar{F} \cap \bar{G} \cap H \cap \bar{A} \cap \bar{B} \cap \bar{C} \cap \bar{D}$	$\mathrm{v} = 0.006435$
VI	Only components e and g killed: $E \cap \bar{F} \cap G \cap \bar{H} \cap \bar{A} \cap \bar{B} \cap \bar{C} \cap \bar{D}$	$\mathrm{vi} = 0.008085$
VII	Only components e and h killed: $E \cap \bar{F} \cap \bar{G} \cap H \cap \bar{A} \cap \bar{B} \cap \bar{C} \cap \bar{D}$	$\mathrm{vii} = 0.006435$
VIII	Only components f and g killed: $\bar{E} \cap F \cap G \cap \bar{H} \cap \bar{A} \cap \bar{B} \cap \bar{C} \cap \bar{D}$	$\mathrm{viii} = 0.006615$
IX	Only components f and h killed: $\bar{E} \cap F \cap \bar{G} \cap H \cap \bar{A} \cap \bar{B} \cap \bar{C} \cap \bar{D}$	$\mathrm{ix} = 0.005265$
X	None of the critical components killed (the corresponding area = A_P minus the sum of the preceding vulnerable areas)	$\mathrm{x} = 52.477016$

Table 11 State transition matrix of probability [T] of example 2

$1/A_P$	Probability of transitioning from this state: I	II	III	IV	V	VI	VII	VIII	IX	X	To this state
	A_P	i + iii + viii + ix	i + vi + ii + vii	i + v + vii + ix	i + iv + vi + viii	i + iii + v + vii + viii + ix	i + iii + iv + vi + viii + ix	i + ii + v + vi + vii + ix	i + ii + iv + vi + vii + viii	i	I
	0	ii + x	0	0	0	0	0	0	0	ii	II
	0	0	iii + x	0	0	0	0	0	0	iii	III
	0	0	0	iv + x	0	0	0	0	0	iv	IV
	0	0	0	0	v + x	0	0	0	0	v	V
	0	iv + vi	0	ii + vi	0	ii + iv + vi + x	0	0	0	vi	VI
	0	v + vii	0	0	ii + vii	0	ii + v + vii + x	0	0	vii	VII
	0	0	iv + viii	iii + viii	0	0	0	iii + iv + viii + x	0	viii	VIII
	0	0	v + ix	0	iii + ix	0	0	0	iii + v + ix + x	ix	IX
	0	0	0	0	0	0	0	0	0	x	X

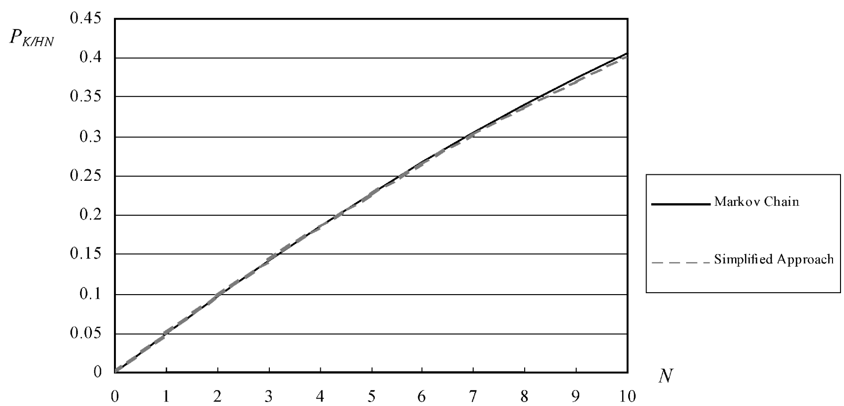

Fig. 3 $P_{\mathrm{K/HN}}$ vs the number of hits from the Markov chain method and the simplified approach.

$$[\boldsymbol{T}] = \begin{bmatrix} 1.000000 & 0.052491 & 0.053020 & 0.050439 & 0.050523 & 0.052723 & 0.052782 & 0.053230 & 0.053284 & 0.050113 \\ 0.000000 & 0.946986 & 0.000000 & 0.000000 & 0.000000 & 0.000000 & 0.000000 & 0.000000 & 0.000000 & 0.002646 \\ 0.000000 & 0.000000 & 0.946505 & 0.000000 & 0.000000 & 0.000000 & 0.000000 & 0.000000 & 0.000000 & 0.002165 \\ 0.000000 & 0.000000 & 0.000000 & 0.944486 & 0.000000 & 0.000000 & 0.000000 & 0.000000 & 0.000000 & 0.000145 \\ 0.000000 & 0.000000 & 0.000000 & 0.000000 & 0.944457 & 0.000000 & 0.000000 & 0.000000 & 0.000000 & 0.000116 \\ 0.000000 & 0.000291 & 0.000000 & 0.002791 & 0.000000 & 0.947277 & 0.000000 & 0.000000 & 0.000000 & 0.000145 \\ 0.000000 & 0.000232 & 0.000000 & 0.000000 & 0.002761 & 0.000000 & 0.947218 & 0.000000 & 0.000000 & 0.000116 \\ 0.000000 & 0.000000 & 0.000265 & 0.002284 & 0.000000 & 0.000000 & 0.000000 & 0.946770 & 0.000000 & 0.000119 \\ 0.000000 & 0.000000 & 0.000211 & 0.000000 & 0.002259 & 0.000000 & 0.000000 & 0.000000 & 0.946716 & 0.000095 \\ 0.000000 & 0.000000 & 0.000000 & 0.000000 & 0.000000 & 0.000000 & 0.000000 & 0.000000 & 0.000000 & 0.944341 \end{bmatrix}$$

We can see that the sum of the elements of each column is equal to 1.000000 except for columns 3 and 10, which can be attributed to the computer errors. The initial state vector prior to the first hit $\{\boldsymbol{S}_0\}$ is

$$\{\boldsymbol{S}_0\} = \{0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 1.0\}$$

After matrix multiplications given by Eq. (5), we have

$$\{S_1\} = \{0.050113, 0.002646, 0.002165, 0.000145, 0.000116, 0.000145, 0.000116, 0.000119, 0.000095, 0.944341\}$$

and

$$\{S_2\} = \{0.097728, 0.005004, 0.004093, 0.000275, 0.000219, 0.000276, 0.000220, 0.000226, 0.000180, 0.891779\}.$$

Hence, the cumulated probabilities the aircraft is killed after the first and second hit are $P_{\mathrm{K/H1}} = \{S_1\}\,[1] = 0.050113$ and $P_{\mathrm{K/H2}} = \{S_2\}\,[1] = 0.097728$, respectively.

3. *Computing the Multiple-Hit Vulnerability Using Simplified Approach*

An approximation for the probability that an aircraft has been killed after N hits can be obtained by the simplified approach.[1] The simplified approach neglects the mutually exclusive feature of the kills of components that do not overlap and assumes that all component kills are independent. In the Markov chain method, if a component in one overlapping area is hit and killed, components not included in that overlapping area cannot be killed, that is, component kills are mutually exclusive for all components outside of the overlapping area hit. In the simplified approach, the assumption is made that all components can be killed on any hit on the aircraft. This approach is considerably simpler than the Markov chain method. However, it might not be accurate enough. According to Ref. 1, using the simplified approach (the binomial approach), the probability the aircraft is killed after N hits is

$$\begin{aligned} P_{\mathrm{K/HN}} = 1 &- \left(1 - P_{\mathrm{k/H}_a}\right)^N \left(1 - P_{\mathrm{k/H}_b}\right)^N \left(1 - P_{\mathrm{k/H}_c}\right)^N \\ &\times \left(1 - P_{\mathrm{k/H}_d}\right)^N \Big[\left(1 - P_{\mathrm{k/H}_e}\right)^N + \left(1 - P_{\mathrm{k/H}_f}\right)^N \\ &- \left(1 - P_{\mathrm{k/He}}\right)^N \left(1 - P_{\mathrm{k/H}_f}\right)^N\Big]\Big[\left(1 - P_{\mathrm{k/H}_g}\right)^N \\ &+ \left(1 - P_{\mathrm{k/H}_h}\right)^N - \left(1 - P_{\mathrm{k/Hg}}\right)^N \left(1 - P_{\mathrm{k/H}_h}\right)^N\Big] \end{aligned} \tag{6}$$

where $P_{\mathrm{k/H}_i}$ is the probability of kill of component i given a hit on aircraft. $P_{\mathrm{k/H}_i}$ is determined by[1]

$$P_{\mathrm{k/H}_i} = \left(A_{p_i} \quad P_{\mathrm{k/h}_i}\right) \big/ A_P \tag{7}$$

Table 12 shows the values for $P_{\mathrm{k/H}_i}$ and A_{p_i}.

Table 12 Values for P_{k/H_i} and A_{p_i}

Component	A_{p_i}, m^2	P_{k/h_i}	P_{k/H_i}
a	0.64	1.00	0.011517
b	1.00	0.40	0.007198
c	$(3.00+0.20+0.80)$	0.25	0.017995
d	$(3.00+0.20+0.80)$	0.20	0.014396
e	$(0.20+0.80)$	0.50	0.008998
f	$(0.20+0.80)$	0.45	0.008098
g	0.20	0.35	0.001260
h	0.20	0.30	0.001080

Figure 3 presents the results for $P_{K/HN}$ vs the number of hits from the Markov chain method and the simplified approach using binomial probability function for the aircraft. Note that $P_{K/HN}$ from the simplified approach is close to the Markov chain value, for this example.

VII. Conclusions

1) In this paper, multiple-hit vulnerability of aircraft with component overlapping when viewed from a specific angle can be calculated by decomposing the overlapping region into subregions whose kill properties are determined from the functional relationship among the overlapping components (e.g., nonredundant, mutually redundant, redundant), and treating the subregions as individual components with appropriate kill properties in Markov chain or tree diagram method.

2) For ease of illustration of the proposed method, this paper has assumed kill of all of the components of the each redundant set lead to kill of the redundant set, but it is not necessary. In vulnerability assessment, when kill of aircraft or redundant sets is expressed as the kill tree, the vulnerable-area decomposition method can be generalized as follows. First, separate the nonredundant components' vulnerable areas out of the overlapping region. Then identify the component kill events according to the combinations of all of the redundant components in the overlapping region. From the kill tree see whether the component kill event will lead to an aircraft kill. If the component kill event leads to an aircraft kill, add the corresponding vulnerable area of the event to the vulnerable area of the aircraft kill state. If not, add the corresponding vulnerable area of the event to the vulnerable area of the relevant no kill state of aircraft. After vulnerable-area decomposition in the overlapping region, the aircraft multiple-hit vulnerability computation can proceed utilizing Markov chain or tree diagram method.

3) The proposed vulnerable-area decomposition method in the components' overlapping region plays a very important role in solving the multiple-hit vulnerability computation of aircraft with multiple redundant sets and arbitrary overlapping among components.

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References

[1]Ball, R. E., *The Fundamentals of Aircraft Combat Survivability Analysis and Design*, 2nd ed., AIAA Education Series, AIAA, Reston, VA, 2003, Chaps. 1 and 5.

[2]Fielding, J. P., and Nilubol, Otsin, "Integraton of Survivability Assessment into Combat Aircraft Design for Operational Effectiveness," *ICAS 2004 Proceedings on Disc* ICAS 2004-1.5.1, Optimage, Ltd., Edinburg, Scotland, U.K., 2004.

[3]Kangalos, A. J., "The Effect of Component Redundancy upon Aircraft Combat Survivability," Master Thesis, Dept. of Aeronautics, Naval Postgraduate School, Montery, CA, Dec. 1983.

[4]Zhang, J., and Wang, H., "An Improved Markov Chain Method for Multiple Hit Vulnerability Assessment," *Journal of Shenyang Institute of Aeronautical Engineering*, Vol. 18, No. 1, 2003, pp. 87–89.

[5]Song, B., and Han, Q., "Theory and Method for Computing Combat Aircraft Vulnerability (II)," *Modern Defense Technology*, Vol. 27, No. 5, 1999, pp. 36–42.

[6]Pei, Y., Song, B., and Han, Q., "Algorithm Study on Geometric Description of Aircraft for Vulnerability Assessment Based on Finite Elements Model," *Journal of Northwestern Polytechnical University*, Vol. 22, No. 5, 2004, pp. 567–571.